On the Difference between Time and Space

Asher Yahalom

Ariel University, Ariel 40700, Israel

Recent Developments in General Relativity

In Memory of Joseph Katz (1930-2016)

May 21-23, 2017

Beit Belgia , Edmond J. Safra Campus, Hebrew University, Jerusalem, Israel

Content

- The difference between time and space
- The stability approach
- Matter included
- The maximal size of space time.
- Metric sign changes
- Implications
- Cosmology and the horizon homogeneity problem.

Bibliography

[1] Asher Yahalom "The Geometrical Meaning of Time" [“The Linear Stability of Lorentzian Space-Time” Los-Alamos Archives - gr-qc/0602034, gr-qc/0611124] Foundations of Physics http://dx.doi.org/10.1007/s10701-008-9215-3 Volume 38, Number 6, Pages 489-497 (June 2008).

[2] Asher Yahalom "The Gravitational Origin of the Distinction between Space and Time" International Journal of Modern Physics D, Vol. 18, Issue: 14, pp. 2155-2158 (2009). DOI: 10.1142/S0218271809016090.

Bibliography

[3] Asher Yahalom "Gravity and the Complexity of Coordinates in Fisher Information" International Journal of Modern Physics D, Vol. 19, No. 14 (2010) 2233–2237, © World Scientific Publishing Company DOI: 10.1142/S0218271810018347.

[4] Asher Yahalom "The geometrical meaning of time - the emergence of the concept of time in the general theory of relativity" a chapter in a book "Advances in Classical Field Theory", Bentham eBooks eISBN: 978-1-60805-195-3,
2011. http://www.bentham.org/ebooks/9781608051953/index.htm.

Bibliography

[5] Yahalom, A. (2013). Gravity and Faster than Light Particles, Journal of Modern Physics (JMP), Vol. 4 No. 10 PP. 1412-1416. DOI: 10.4236/jmp.2013.410169.

[6] Asher Yahalom "On the Difference between Time and Space" Cosmology 2014, Vol. 18. 466-483. Cosmology.com.

[7] Asher Yahalom "The Geometrical Meaning of Time - Some Cosmological Implications" Proceedings of 3rd International Conference on Mathematical Modeling in Physical Sciences (IC-MSQUARE 2014), 28–31 August 2014, Madrid, Spain. Journal of Physics: Conference Series (IOP Publishing), Volume 574, 012061, 2015. doi:10.1088/1742-6596/574/1/012061.

Bibliography

[8] A. Yahalom “The Geometrical Meaning of Time in the Presence of Matter” Journal of Physics: Conference Series 633 (2015) 012030 doi:10.1088/1742-6596/633/1/012030 IOP Publishing.

[9] Asher Yahalom “Preliminary Stability Analysis of a Friedman-Lemaitre-Robertson-Walker Universe” Accepted to the Proceedings of the 10th Biennial Conference on Classical and Quantum Relativistic Dynamics of Particles and Fields, 6 - 9 June 2016, Ljubljana, Slovenia.

[10] Asher Yahalom “Gravity, Stability and Cosmological Models” – Gravity research foundation honorable mention (2017).

The difference between time and space

It is well known that our daily space-time is approximately of Lorentz (Minkowski) type that is, it possesses the metric $\eta_{\mu\nu}$ = diag (1,-1,-1,-1). The above statement is taken as one of the central assumptions of the theory of special relativity and has been supported by numerous experiments.

But why should it be so?

Hermann Minkowski

1864 - 1909

“The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.”

Minkowski's address delivered at the 80^{th} Assembly of German Natural Scientists and Physicians (21 September 1908)

What does it mean a “kind of union”?

And if united why are space and time different?

One may reply, well that is the way things are, time and space are obviously different, why should we bother ourselves?

But this is not a scientific approach!!

For instance the mass of the electron is known empirically to a reasonably high accuracy but this does not explain why electrons have such a mass nor does it explain why electrons exist at all.

To explain such phenomena researchers try to construct theories such as string theory which hopefully will yield an explanation.

Thus well established empirical facts demand theories to explain them. To put this in other words, science is not just a collection of well established facts, rather it is a struggle to explain those well established facts using a minimal number of assumptions.

The emphasis is on the word minimal because explaining well established empirical facts with an arbitrary number of assumptions is not a challenge at all.

William of Ockham

1285 - 1347

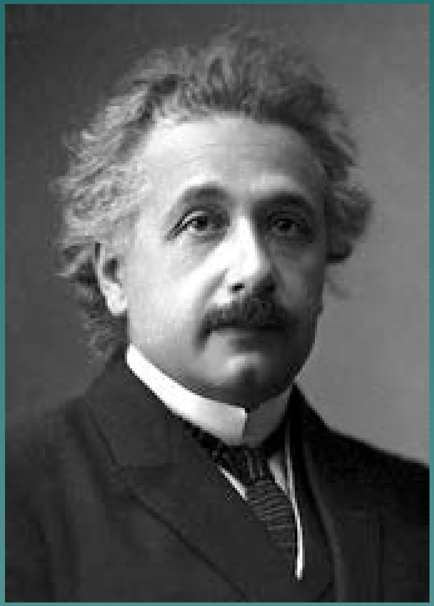
Ockham’s Razor:

“Plurality is not to be posited without necessity.”

Don’t multiply complex causes to explain things when a simple one will do

Albert Einstein

1879 - 1955



“It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience.”

“On the Method of Theoretical Physics” the Herbert Spencer Lecture, Oxford, June 10,1933.

Did Albert Einstein practiced his preaching in the construction of the General Theory of Relativity or could he do better?

Most textbooks (and also Joseph Katz lecture notes of the beginning of the 1990's) state that in the general theory of relativity any space-time is locally of the type:

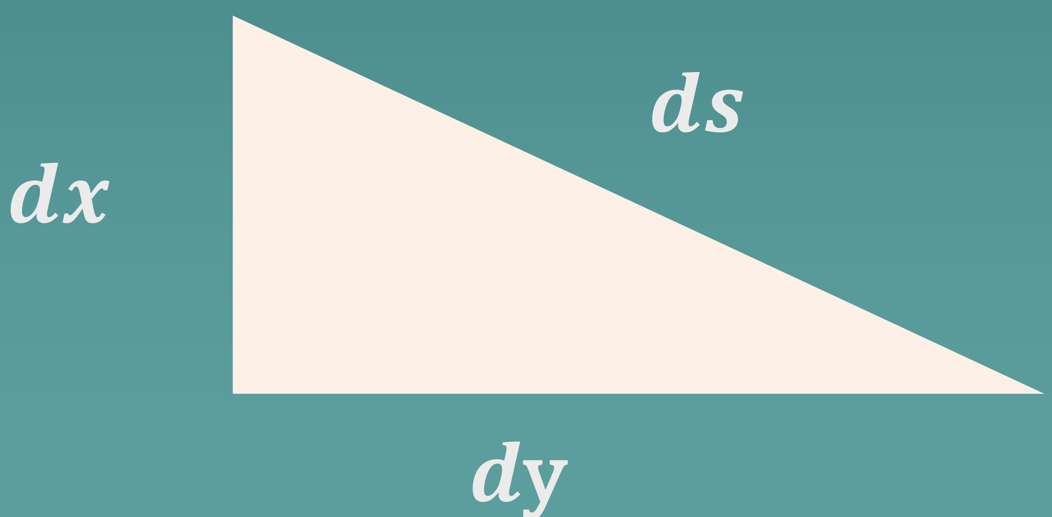
$$\eta_{\mu\nu} = \mathrm{diag}\,(1,-1,-1,-1)$$

although it can not be presented so globally due to the effect of matter. This is a part of the demands dictated by the well known equivalence principle.

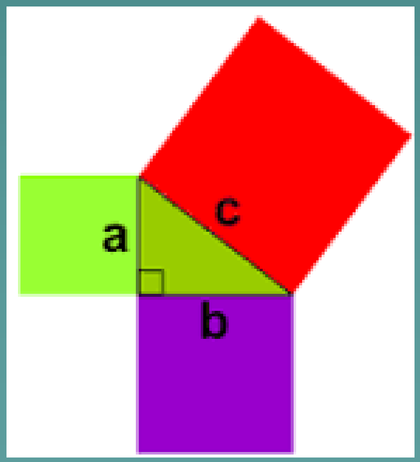
Riemannian Geometry is replaced by Pseudo Riemannian Geometry.

In the case of Riemannian Geometry:

$$ds^2 = dx^2 + dy^2$$

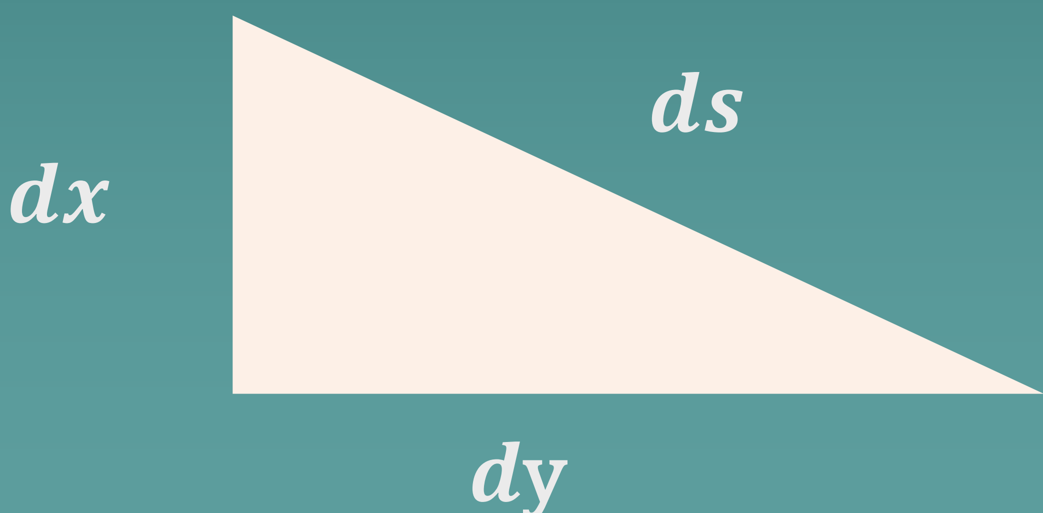


Pythagoras' theorem



In the case of our Pseudo Riemannian space time something strange happens:

$$ds^2 = dx^2 - dy^2$$



Why?

Because we know that nature obeys (locally) the rules of special relativity for "empty" space-time.

Wrong answer!

(Equivalent to saying that things are what they are because that is how things are)

Never the less the above principle is taken to be one of the assumptions of general relativity other assumption such as diffeomorphism invariance, and the requirement that theory reduce to Newtonian gravity in the proper regime lead to the Einstein equations:

$$G_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

Can we reduce the number of assumptions as Einstein recommends?

The stability approach

At about the same time as I was taking Joseph Katz course in General relativity I was also doing my MSc followed by a PhD under the supervision of Joseph in the stability of stationary self gravitating flows (Galactic Models).

It then occurred to me that the question of Quasi Riemannian geometry may be connected to the notion of stability.

It took me more than ten years and a Sabbatical in Cambridge to be able to formulate the problem mathematically.

The stability approach

Let us look at the possible constant metrics available in the general theory of relativity which are not equivalent to one another by a trivial transformation, that amounts to a simple change of coordinates.

Since the metric is a symmetric matrix we can diagonalize it using a unitary transformation in which both the transformation matrix and the eigenvalues obtained are real. Thus without loss of generality we can assume that in a proper coordinate basis:

$$\eta = \mathrm{diag}\left(\lambda_0, \lambda_1, \lambda_2, \lambda_3\right).$$

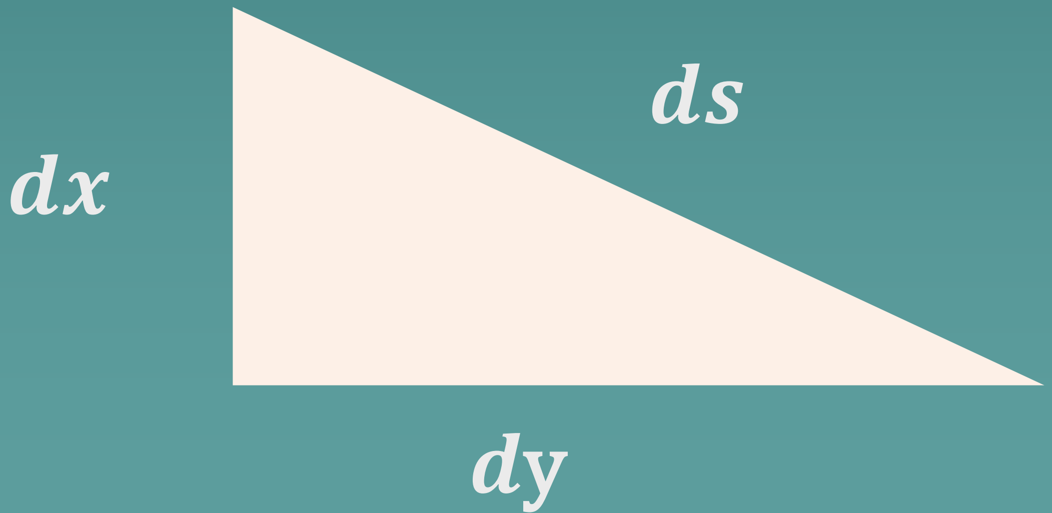
By changing the units of the coordinates (scaling), we can always obtain:

$$\eta = \mathrm{diag}\,(\pm 1, \pm 1, \pm 1, \pm 1)$$

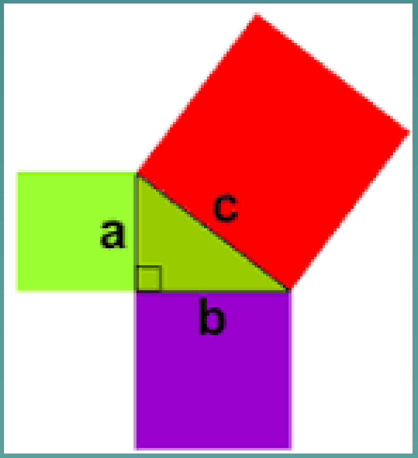
Notice that a zero eigen-value is not possible due to our assumption that the space is four dimensional.
All those metrics are perfectly good solutions of Einstein Equations for a flat space.

So the only real freedom is in the signs of the diagonal elements. In the case of an Euclidian space-time all signs are positive hence:

$$ds^2 = dx^2 + dy^2$$

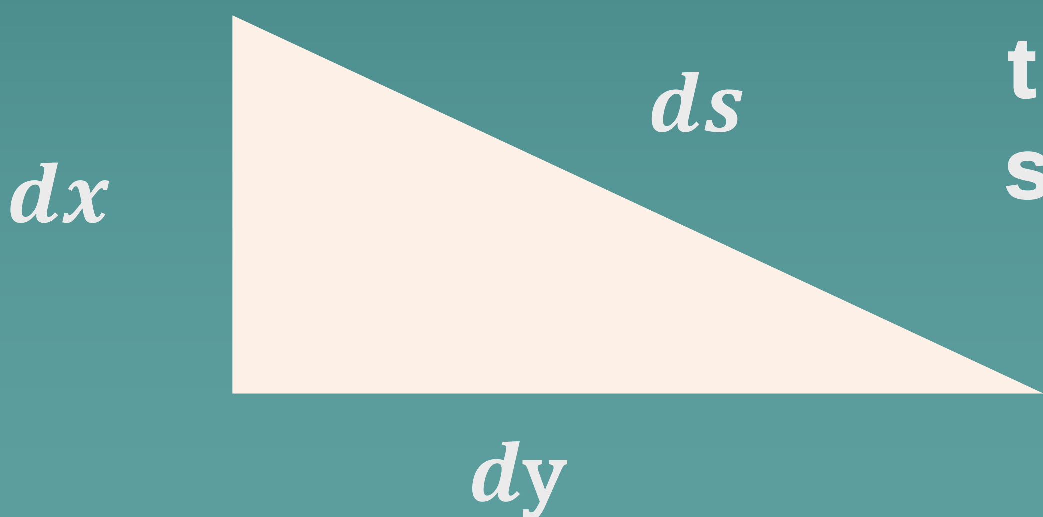


Pythagoras' theorem



In the case of our <u>real</u> space-time something strange happens:

$$ds^2 = dx^2 - dy^2$$



Why does nature chooses this absurd solution?

The empty space case

In the lack of matter Einstein Equations become:

$$G_{\mu\nu} = 0$$

This "almost" always true as the left hand side is small unless for extreme cases (black holes etc.).
We make a small perturbations to any flat space time in order to study its stability:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

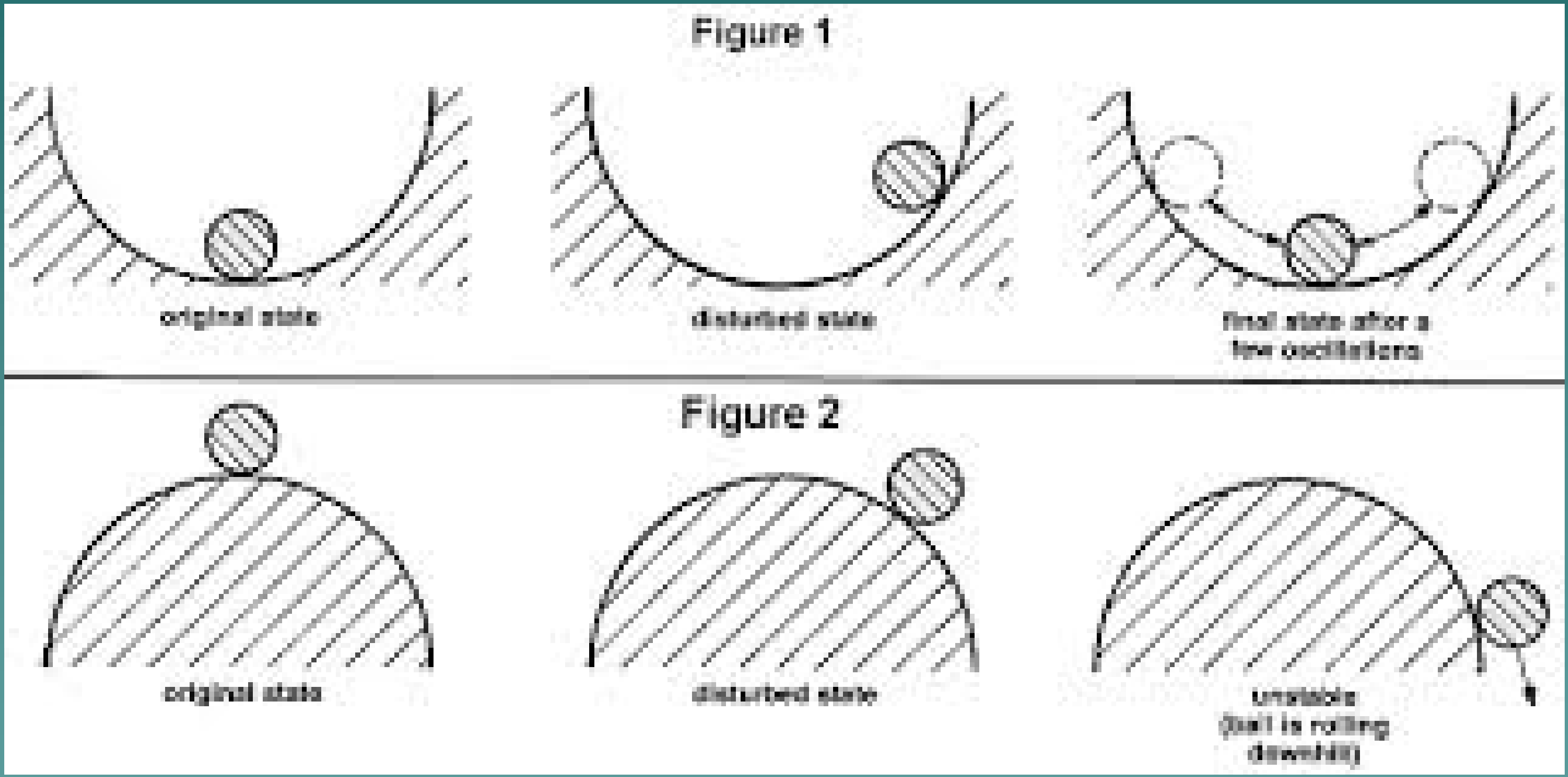


Figure 1
original state
disturbed state
final state after a
few oscillations
Figure 2
original state
disturbed state
unstable
(ball is rolling
downhill)

We define:

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$$

Now we can write the Einstein equations in terms of this expression (keeping first order terms):

$$\bar{h}_{\mu\nu,\alpha}{}^{\alpha} = 0$$

In which we assumed a proper gauge.

We solve the above equation using Fourier decomposition:

$$\bar{h}_{\mu\nu} = \frac{1}{(2\pi)^{\frac{3}{2}}} \int_{-\infty}^{\infty} A_{\mu\nu}(x_0, \vec{k}) e^{i\vec{k}\cdot\vec{x}} d^3k$$

$$\vec{k} = (k^1, k^2, k^3), \quad \vec{x} = (x^1, x^2, x^3)$$

Which take the form:

$$\eta^{00}\partial_0^2 A_{\mu\nu} - \eta^{mn}k_m k_n A_{\mu\nu} = 0$$

Choosing: $\eta^{00} = 1$

We see that the only way to avoid exploding solutions is to choose :

$$\eta^{mn} = \mathrm{diag}\,(-1,-1,-1)$$

Choosing: $\eta^{00} = -1$

We see that the only way to avoid exploding solutions is to choose :

$$\eta^{mn} = \text{diag}\ (1, 1, 1)$$

Hence the only <u>stable</u> type of metric is the Lorentz type.

The existence of time is not an assumption of general relativity it is a result of general relativity!

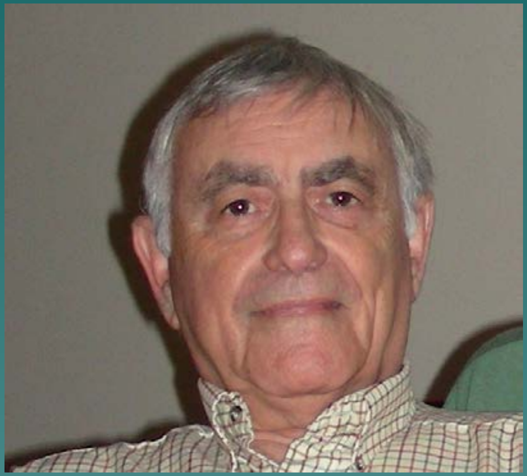
Question: why should we not assume "boundary conditions" that will take care of the unstable perturbation even in the Euclidean case?

Answer 1: How should nature know to impose such boundary conditions in every point of space-time?

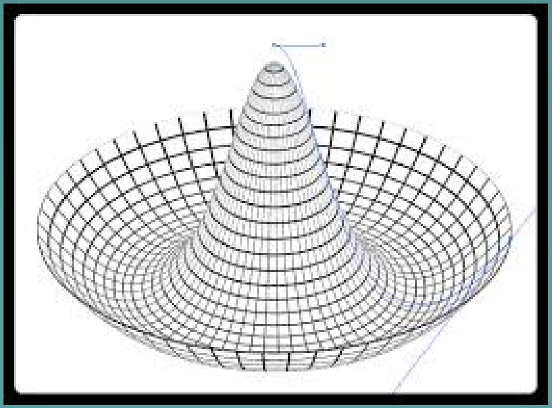
Answer 2: Special boundary conditions is an additional assumption. Additional assumptions is something we are trying to avoid.

Joseph Katz

1930-2016



Question: Linear stable configurations can be nonlinearly unstable and linear unstable configurations can nonlinearly stable. So is linear stability analysis sufficient?



Answer 1: Lorentz metric is stable also non-linearly. See Christodoulou, D. & Klainerman, S. (1989-1990). The global nonlinear stability of the Minkowski space, Seminaire Equations aux derivees partielles (dit "Goulaouic-Schwartz"), Exp. No. 13, p. 29.

Answer 2: Non-linear general relativity is only significant in extreme cases (big bang, black holes etc..) this is very remote from the empty space scenario under investigation. (my thanks to Donald for pointing this out).

Answer 3: As the question of non-linear stability of Lorentz space-time is settled all we have to do as Joseph pointed out is to prove nonlinear instability for a single type of perturbation, work under progress…

Matter included

So far we have discussed the case of stability of the Lorentzian metric in empty space.

The result obtained is rather philosophical, but does it have any physical consequences?

We expect that this result should be valid also to the case of an almost empty space which is the generic situation but we cannot be sure unless we check.

Let us assume a fluid energy momentum tensor of the form:

$$T_{\mu\nu} = (p + \rho c^2) u_\mu u_\nu - p g_{\mu\nu}$$

In the above p is the pressure, ρ is the density and $u_\mu = \frac{dx_\mu}{ds}$, in which the interval ds is defined as:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

We notice that $T_{\mu\nu}$ depend on metric perturbations both directly through the term $-p\, g_{\mu\nu}$ and through the ds term.

If we assume a uniform density then we have a Friedman-Lemaitre-Robertson-Walker metric.

$$g^{(0)}_{\mu\nu} = \mathrm{diag}\ \left(1, -\frac{S^2(t)}{1-kr^2}, -S^2(t)r^2, -S^2(t)r^2\sin^2\theta\right)$$

Perturbation leads to the following equations:

$$R^{(1)}_{ab} - g^{(1)}_{ab}\frac{4\pi G}{c^4}(\rho c^2 - p) = 0, \qquad R^{(1)}_{\mu 0} + g^{(1)}_{\mu 0}\frac{4\pi G}{c^4}(\rho c^2 + 3p) = 0$$

a,b are spatial indices.

In the presence of matter stability analysis involves a critical wave number (in the case that pressure is negligible with respect to density) :

$$k > k_{crit} = \frac{4}{c}\sqrt{2\pi G\rho}$$

In terms of wavelengths we see that an upper scale for stable perturbations is:

$$\lambda < \lambda_{max} = \frac{2\pi}{k_{crit}} = \frac{c}{2}\frac{\sqrt{\pi}}{\sqrt{2G\rho}}$$

The maximal size of space time

The density of the universe is estimated to be (”Universe 101: What is the Universe Made Of?”. NASA: WMAP’s Universe. Jan 24, 2014. Retrieved 17/2/2015).

$$\rho \simeq 4.5 \cdot 10^{-28} \frac{Kilogram}{Meter^3}$$

Which leads to:

$$\lambda_{max} \simeq 1.08 \cdot 10^{27} Meter$$

This is slightly larger than the radius of the observable universe:
(Bars, I. & Terning, J. (2009). Extra Dimensions in Space and Time. Springer. pp. 27. ISBN 978-0-387-77637-8. Retrieved 2011-05-01).

$$14 \cdot 10^{9} Parsec \simeq 4.32 \cdot 10^{26} Meter$$

Intermediate Conclusion

Hence in ”surprising” coincidence the diameter of the observable universe is about the size of the largest scale stable perturbation. At this time the size of the universe is not known but it is suspected that above the stability scale the metric of space-time and hence physics may be quite different.

Simple Perturbations

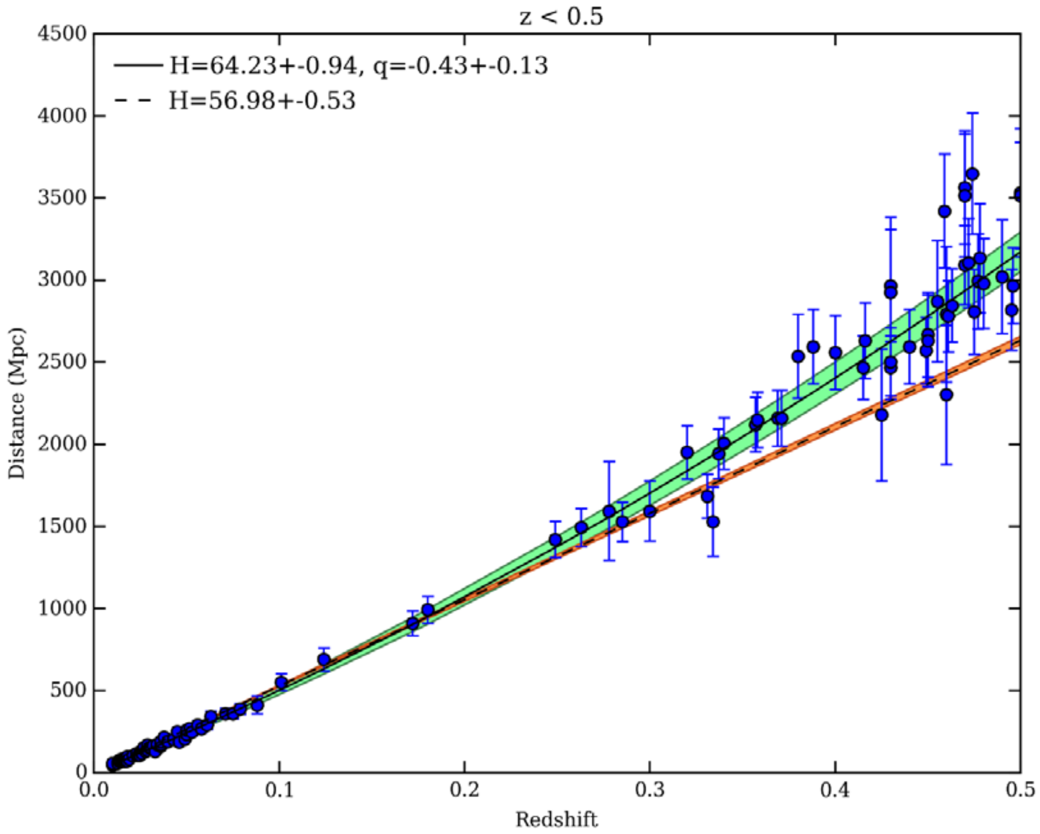
Friedman equation

$$\frac{\dot{S}^2 + kc^2}{S^2} = \frac{8\pi G\rho_0}{3}\frac{S_0^3}{S^3}$$

$$\delta S(t)$$

$$\frac{\lambda(t_2)}{\lambda(t_1)} = \frac{S(t_2)}{S(t_1)} = 1 + z$$

$$\delta z = (1+z)\left(\frac{\delta S(t_2)}{S(t_2)} - \frac{\delta S(t_1)}{S(t_1)}\right)$$

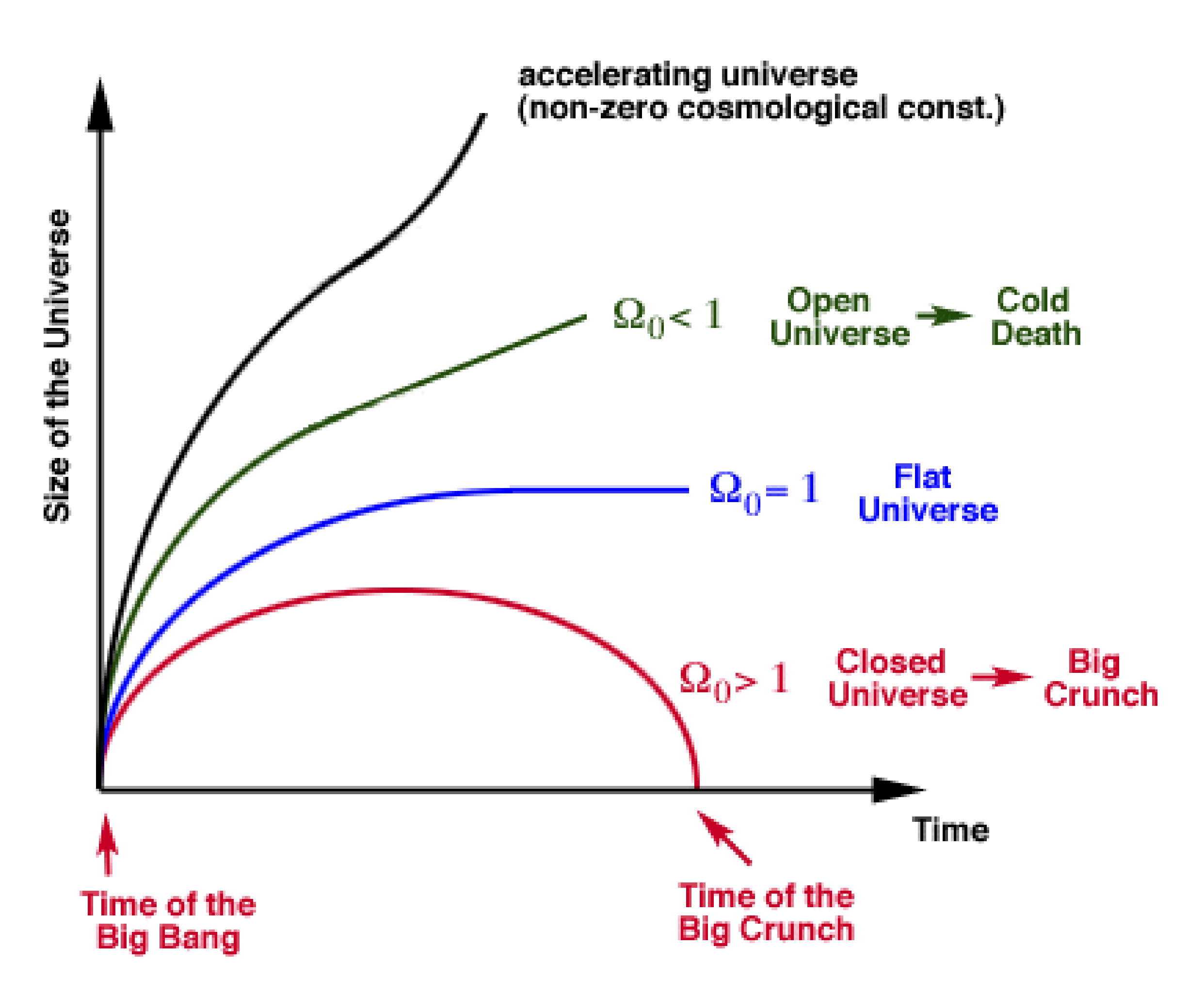


http: / / adamdempsey / ygrene_krad / nohtyp / oi.buhtig.90 atadeht#lmth.ygrene_krad

$$\delta \dot{S} = -\frac{4\pi G \rho_0 S_0^3}{3 S^2 \dot{S}} \delta S \equiv -\tilde{\lambda}(t) \delta S$$

$$\delta S(t) = \delta S(t_0) e^{-\int_{t_0}^{t} \tilde{\lambda}(t') dt'}$$

Hence the perturbation will decay provided that $\tilde{\lambda} > 0$, however, this will only happen for and expanding universe ($\dot{S} > 0$).


accelerating universe
(non-zero cosmological const.)
Size of the Universe
$\Omega_0 < 1$
Open Universe
Cold Death
$\Omega_0 = 1$
Flat Universe
$\Omega_0 > 1$
Closed Universe
Big Crunch
Time
Time of the Big Bang
Time of the Big Crunch

A Cosmological Constant

$$G_{\mu\nu} + \lambda g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\frac{\dot{S}^2 + kc^2}{S^2} - \frac{1}{3}\lambda c^2 = \frac{8\pi G \rho_0}{3} \frac{S_0^3}{S^3}$$

$$\delta\dot{S} = -\frac{1}{3}c^2 \frac{S}{\dot{S}}\left(\lambda_c \frac{S_0^3}{S^3} - \lambda\right)\delta S \equiv -\tilde{\lambda}_2(t)\delta S.$$

$$\lambda_c \equiv \frac{4\pi G \rho_0}{c^2}$$

$$\delta S(t) = \delta S(t_0) e^{-\int_{t_0}^{t} \tilde{\lambda}_2(t') dt'}$$

Conditions of Stability:

$$\begin{cases} \lambda < \lambda_c \frac{S_0^3}{S^3}, & \dot{S} > 0; \\ \lambda > \lambda_c \frac{S_0^3}{S^3}, & \dot{S} < 0. \end{cases}$$

$$\Omega_\Lambda < \frac{1}{2}\Omega_m, \qquad \Omega_\Lambda \equiv \frac{\lambda\, c^2}{3H_0^2}, \quad \Omega_m \equiv \frac{\rho_0}{\rho_c}$$

In which $H_0 = \left(\frac{\dot{S}}{S}\right)_0$ is Hubble's constant at the present epoch and the critical density is $\rho_c = \frac{3H_0^2}{8\pi G}$. The current values of the parameters [13] are $\Omega_\Lambda = 0.6911 \pm 0.0062$ and $\Omega_m = 0.3089 \pm 0.0062$.

Hence the Lambda-CDM model is not stable

Some more perturbations

$$R^{(1)}_{ab} - g^{(1)}_{ab}\frac{4\pi G}{c^4}(\rho c^2 - p) = 0, \qquad R^{(1)}_{\mu 0} + g^{(1)}_{\mu 0}\frac{4\pi G}{c^4}(\rho c^2 + 3p) = 0$$

$$h \equiv g^{(1)}_{11}(t,r)$$

$$\frac{1}{2}\partial_0^2 h + \frac{(1-kr^2)}{rS(t)^2}\partial_1 h + \frac{(S'(t)^2 - 2c^2 k)}{c^2 S(t)^2}h - \frac{4\pi G}{c^2}\rho h = 0$$

in case pressure is negligible with respect to ρc^2

$$\partial_t^2 h + \frac{2c^2\left(1-kr^2\right)}{rS(t)^2}\partial_1 h + 2\left(H^2(t) - 2c^2\frac{k}{S(t)^2} - 4\pi G\rho\right)h = 0$$

$$\partial_t^2 h + 2\left(H^2(t) - 2c^2\frac{k}{S(t)^2} - 4\pi G\rho\right)h = 0$$

Neglecting radial perturbations we obtain the stability condition:

$$H^2(t) - 2c^2\frac{k}{S(t)^2} - 4\pi G\rho > 0$$

$$\frac{2}{3}\rho_c - c^2\frac{k}{2\pi G S_0^2} - \rho_0 > 0$$

This is indicates that an Euclidean section universe with $k = 0$ is unstable since it requires $\rho_0 = \rho_c$, the same can be said about a universe with a closed section in which $k = 1$. This universe requires $\rho_0 > \rho_c$, this clearly cannot be satisfied at the same time

The open section universe with $k = -1$ is stable to this type of perturbation provided that its density satisfies:

$$\frac{2}{3}\rho_c + \frac{c^2}{2\pi G S_0^2} > \rho_0$$

Since $\rho_c = 1.83 \cdot 10^{-26} \frac{Kg}{M^3}$ while ρ_0 is estimated to be [17] $4.5 \cdot 10^{-28} \frac{Kg}{M^3}$, the stability condition for this type of perturbation is satisfied. Thus the current stability analysis is consistent with observations of an expanding universe.

Conjecture

High density regions of space time may support metrics with different signatures with respect to Lorentz.

Metric sign changes

So far we have discussed the stability of a Lorentzian Space-Time, but are there solution of GR in the metric is not Lorentz in some part of space time?
In other words are there metric sign changes?
And if so what are the physical implications of this?

A Black Hole

The Schwarzschild square interval (in terms of spherical coordinates) is given by:

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_s}{r}} - r^2 (d\theta^2 + \sin^2\theta \, d\phi^2)$$

In which the Schwarzschild radius is given by:

$$r_s = \frac{2GM}{c^2}$$

For $r > r_s$ we have a local metric of the type:

$$g_{\mu\nu} = \text{diag}\,(+1, -1, -1, -1)$$

Which is a Lorentz type metric. For $r < r_s$ we have a local metric of the type:

$$g_{\mu\nu} = \text{diag}\,(-1, +1, -1, -1)$$

Which is also a Lorentz type metric.
But notice the shocking exchange of space and time! Signs have change but not the nature of the metric.

Implications

Let us look at the action:

$$\mathcal{A} = \int L d\tau, \qquad L = \frac{1}{2} m u_\alpha u^\alpha + \frac{q}{c} u_\alpha A^\alpha$$

$$u_\alpha \equiv \frac{dx_\alpha}{d\tau}$$

And assume a general constant metric. This action implies the following set of equations:

$$m \frac{du^\alpha}{d\tau} = -\frac{q}{c} u^\beta (\partial_\beta A^\alpha - \partial^\alpha A_\beta)$$

τ is a parameter along the trajectory usually defined as the absolute value of the interval (the proper time):

$$d\tau^2 = |\eta_{\alpha\beta} dx^\alpha dx^\beta|$$

The subluminal Lorentz case:

At t=0 the particle is subluminal.

$$d\tau^2 = c^2 dt^2 (1 - \frac{v^2}{c^2}), \qquad d\tau = c dt \sqrt{1 - \frac{v^2}{c^2}}$$

$$\frac{d}{dt}\left(m \frac{\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}}\right) = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

This is the standard case. A particle which is subluminal will always stay subluminal.

The superluminal Lorentz case:

At t=0 the particle is superluminal.

$$d\tau^2 = c^2 dt^2 (\frac{v^2}{c^2} - 1), \qquad d\tau = c dt \sqrt{\frac{v^2}{c^2} - 1}$$

$$\frac{d}{dt}\left(m \frac{\vec{v}}{\sqrt{\frac{v^2}{c^2} - 1}} \right) = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

This is a non standard case. A particle which is superluminal will always stay superluminal.

The Euclidean case:

$$d\tau^2 = c^2 dt^2 (1 + \frac{v^2}{c^2}), \qquad d\tau = c dt \sqrt{1 + \frac{v^2}{c^2}}$$

$$\frac{d}{dt}\left(m \frac{\vec{v}}{\sqrt{1 + \frac{v^2}{c^2}}} \right) = q \left(\vec{E} - \vec{v} \times \vec{B} \right)$$

A particle is indifferent to whether it is superluminal or subluminal and its velocity can cross the speed of light from either direction.

Consider the following scenario:

A particle is accelerated to a velocity close to the velocity c in a Lorentz space, enters into an Euclidean space and accelerated further in this region to velocities above the speed c and emerge in a Lorentz space in which it will remain above the speed c for ever unless it is decelerated in an Euclidean space again.

The homogeneity (horizon) problem

According to an analysis given by Narlikar a proper radius for a particle horizon of a sub luminal particle at the radiation dominated epoch was $R_L = 2ct$, taking into account temperatures of the early universe led him to conclude that this radius was of the order of magnitude of about 1 meter on present day scales.

In reality the cosmological micro wave background is homogeneous on a scale of 10^{26} meters.

Notice, however, that superluminal particles are not restricted by the velocity of light and hence can bring a very young universe into thermal equilibrium.

Conclusion

1. Time should not be assumed, it should be derived (the direction of time was not discussed but is the result of the H-theorem combined with the low entropy conditions at the big-bang)
2. Space-time has a finite radius, above of which physics is quite different.
3. A possible explanation to the homogeneity of the cosmic microwave background (horizon problem) is suggested. Of course many details of this explanation need to be worked out.

Conclusion

4. A more popular explanation is the “inflation” model suggested by Alan Guth, postulating one or more scalar fields which are needed in order to provide an explanation.
5. The original model of Guth suffered from an entropy problem (predicted too much entropy). Later suggestions by Linde suffered from the need to fine tune the parameters.
6. Moreover, it was shown that a possible explanation within the frame-work of standard Cosmology does exist for the horizon problem if one looks closely at the metric changes of the Friedman-Lemaitre-Robertson-Walker metric.

Conclusion

7. **A basic flaw in common to all inflation models. All inflation models require to postulate one or more scalar fields which have no function, implication or purpose in nature except for their ad-hoc use in the inflation model. This is in sharp contradiction with the principle of Ockham's razor which demand that a minimum number of assumptions will explain a maximum number of phenomena. Postulating a physical field for every phenomena does not serve the purpose of theoretical science.**

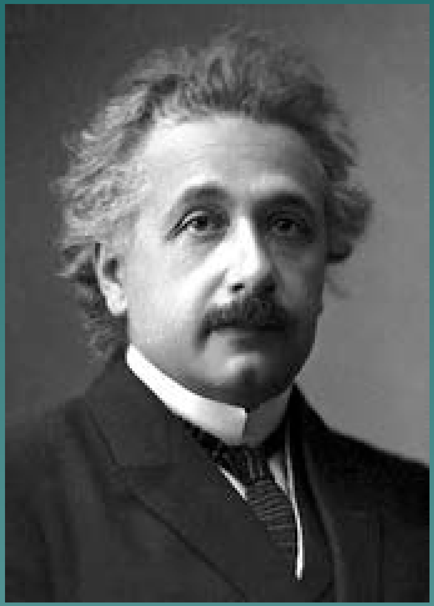
William of Ockham

1285 - 1347

lex parsimoniae

Albert Einstein

1879 - 1955



"It is not the result of scientific research that ennobles humans and enriches their nature, but the struggle to understand while performing creative and open minded intellectual work."

Mein Weltbild, 1934, 14.