Thermodynamics and Gravitation

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Recent Developments in General Relativity
In Memory of Joseph Katz (1930-2016)
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outline

- My connection with Joseph
    - Common collaborators
    - Common interests
- My “interactions” with Joseph
    - Common interests
- Density of states for black hole systems
- Three ongoing problems
    - Boundary conditions for rotating (Kerr) black holes in a cavity
    - Waveforms for LISA
    - Black hole evaporation

My connection with Joseph Katz

- Common colleagues
  - Lynden-Bell brought him to Cambridge (1977-2015)
  - Greg Comer brought me to Jerusalem (1991)
  - Marc Henneaux and Journées Relativiste in Bruxelles (1993)
  - Manuscript with Gerry Horwitz using Katz variables (1995)
- Common interests
  - Thin shells and gravitational collapse
  - Black holes and thermodynamics
  - Variational principles and conserved quantities

My interactions with Joseph Katz

- Not a collaborator
  - not a manuscript, not even a photograph
- Fertility of ideas
  - willingness to consider the unusual
- Passionate about principles
- Adamant about methods
- A true colleague
  - could agree to disagree
- A long-lived inspiration

Matter Shells

- Oscillating shells: Brady, Louko and Poisson
- Thick Shells: Comer and Katz
- Junction conditions: Goldwith and Katz
- Collapsing matter shells: Hajicek and collaborators
- Collapsing null shells: Louko, Whiting and Friedman
- Static shells and solenoids: Lynden Bell and Katz

Black hole in a cavity

- Whiting and York - Pure Gravity
  - Boundary condition description
- Katz and collaborators
  - Thermodynamics and Gravitation
- Simple formulation for spherical symmetry
- What shape cavity for a rotating black hole?
  - This is apparently not an elementary problem

Variational Principles

- Emphasis on boundary conditions (but not Katz terms?)
- Extension to mini-superspace models
  - No substitute for QFT calculations in curves ST
- Path integrals formulations
  - Foundations of Physics (cf Katz)
- Contour integral freedom (cf cosmology)
  - Possibly complex for finite results
  - Role for Wheeler - de Witt equation
- Lagrangian and Hamiltonian approaches
  - Every integral in every step made finite

Density of States

- Incorporates variational principles, thin shells and thermodynamics
- Follows inspiration from Joseph Katz and quantum cosmology
- Louko and Whiting (Note: $M=E(1-E/2r_0)$):

$$\nu(E, r_0) = \begin{cases} 0 & \text{for } E < 0, E > 2r_0 \\ \frac{1}{\pi} \int\limits_0^{2E - \frac{E^2}{r_0}} \mu(r_+) dr_+ \frac{\exp(\pi r_+^2)}{\sqrt{r_0(r_0 - r_+) - (E - r_0)^2}} & \text{for } 0 \leq E \leq 2r_0 \end{cases}$$

- Melmed and Whiting:

$$\nu(E, r_0) = \int\limits_0^{\infty} \frac{d\alpha}{\sqrt{\alpha r_0}} \exp\left(2\pi\alpha E - \pi\alpha\frac{E^2}{r_0} - \frac{\pi\alpha^2}{4}\right)$$

$$= \frac{(2\pi)^{(1/4)}}{\sqrt{r_0}} D_{-1/2}\left(-\sqrt{8\pi}E\left(1 - \frac{E}{2r_0}\right)\right) \exp\left(2\pi E^2\left(1 - \frac{E}{2r_0}\right)^2\right)$$

- Path integral approach, Wheeler - de Witt equation, etc.

Static and stationary black holes - very different

- In GR, mass enters as an integration constant, angular momentum as a coefficient in a solution to a pre (cf also spheroidal coordinates)
- Non-rotating black hole in a spherical cavity can be given a Hamiltonian formulation (Louko and Whiting)
- Amplitude given in terms of cavity size and time interval:

$$K(\mathbf{m}; T_B; \Theta_H) = \exp\left(-ir_0\left(1 - \sqrt{1 - \frac{2\mathbf{m}}{r_0}}\right) T_B + 2i\mathbf{m}^2\Theta_H\right)$$

- In stationary systems, angular momentum easily formulated for finite cavities (Komar)
- What shape should cavity have to allow a variational problem “find” the black hole parameters of system?

Recent developments? The last forty years

- Joseph first visited Cambridge in 1977
- Bekenstein and Hawking had already established entropy and temperature for (quantum) black holes
- Thermal quantum states were being considered for equilibrium and evaporating black holes (Christensen, Candelas)
- Require expectation values of stress energy tensor for quantum fields states in black hole space-times
- Symmetry principles reduce problem to one unknown component
- Fawcett and Whiting had not yet begun numerical calculations
- Forty years on - currently only numerical calculations available

singularities in quantum field theory

- products of fields at same space-time event
- typically use a Green's function as a proxy
- singularity understood in terms of Hadamard expansion
- point splitting allows for regularization and subtraction to avoid infinities
- calculations based on mode sum representations
- results are different for different vacuum states

singularities in the self force problem (eLISA calculations)

- arise from modeling a small mass as a point source
- point represented by an integrated Dirac delta function source along its entire world-line
- can integrate against Green's function to find solution
- regularization gives smooth (locally vacuum) residual solution in neighborhood of source
- subtraction lies outside future light cone and outside past light cone
- its necessary to regularize both the gravitational field (metric) and its derivative

evolution in self force calculations

- particle radiates as it moves along “geodesic”
- need to adjust orbit as the particle moves in time
- need a method capable of evolving over 100,000 orbits, but first order perturbation analyst not enough
- two-time schemes are under development
- need to go to second order in perturbation analysis
- several schemes, but no calculation so far

space-time evolution in the presence of Hawking Radiation

- typically, equilibrium (plus corrections) state used to define quantum stress tensor
- physical arguments used to determine some space-time symmetries of quantum source (spherical, conserved, etc)
- quantum arguments used to determine trace anomaly, etc
- have one more unknown than equations
    - this situation has not changed in almost forty years
- different authors make different choice about how to deal with this missing information, unless they use exact numerical data for the stress tensor
- don't have any exact results for the quantum state during evaporation
- BMS states on horizon do not seem to add enough new information

space-time evolution in the presence of Hawking Radiation

- need to understand quantum state properties better than we do - can physical intuition help in ways we have missed so far?
- we only have numerical solutions with which to describe quantum states accurately
- series expansions (eg Leonard Parker) have not helped to inspire a better understanding
- can't separately characterize the dependence of the state on the ingoing and the outgoing radiation
- what can we understand about vacuum polarization in a curved geometry?
- can we use Parker type analysis to explore the black hole problem further?