# Formation and evaporation of black holes without singularity

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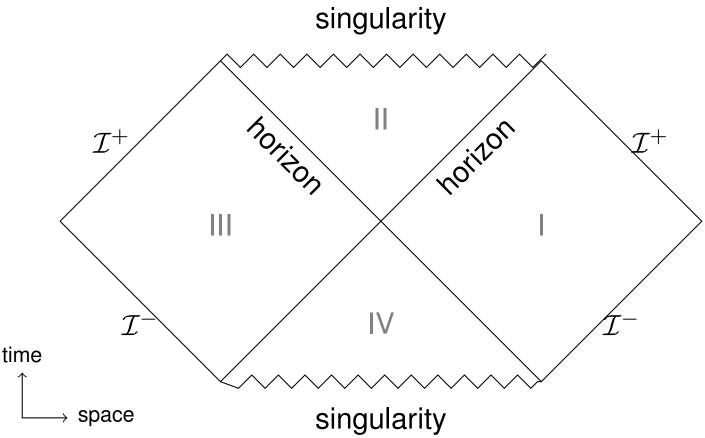
# Schwarzschild's black hole (1915) [1]

$$ds^2 = -\left(1 - \frac{2M}{R}\right) dt^2 + \left(1 - \frac{2M}{R}\right)^{-1} dr^2 + R^2 d\Omega^2$$

- Schwarzschild's metric : exact **static** solution of Einstein equations in vacuum
- This solution describes the exterior of a star of mass $M$ which is spherically symmetric
- A star with a radius $R < 2M$ is actually a **black hole**, i.e. a region where gravity is so strong that neither matter nor light can escape, which also has a singularity at $R = 0$.

1. $c = G = 1$

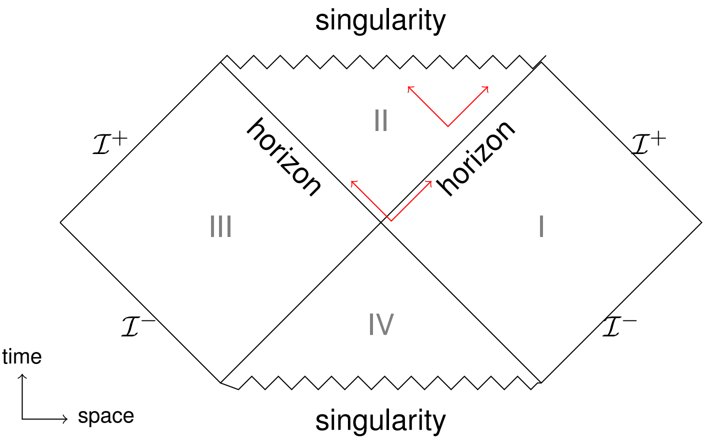


singularity
II
horizon
horizon
$\mathcal{I}^+$
$\mathcal{I}^+$
III
I
$\mathcal{I}^-$
$\mathcal{I}^-$
IV
time
space
singularity

# Schwarzschild's black hole (1915)

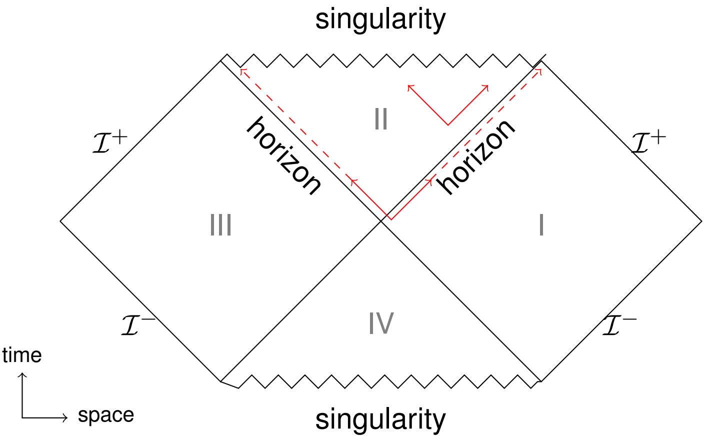
## Penrose diagram of Schwarzschild's space-time





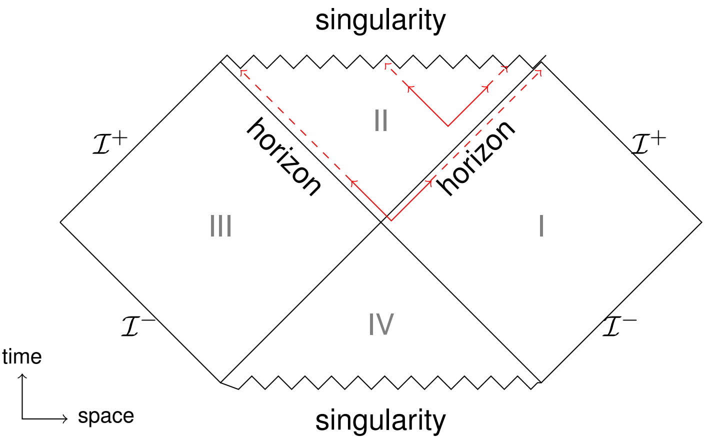
# Schwarzschild's black hole (1915)

## The event horizon, a global concept



# Schwarzschild's black hole (1915)

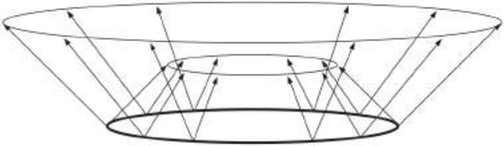
## The event horizon, a global concept




$$\mathcal{B} = \mathcal{M} - I^-(\mathcal{I}^+)$$
$$\mathcal{H} = \partial\mathcal{B}$$

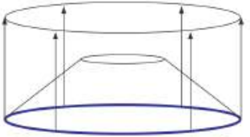
# Trapped surface : a local concept

## Notion of apparent/trapping horizon [2]



**Untrapped** surface

**Trapped** surface



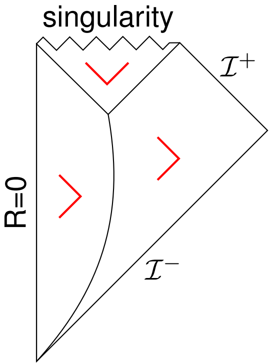
**Marginal** surface : apparent horizon

Apparent horizons are suited to astrophysics

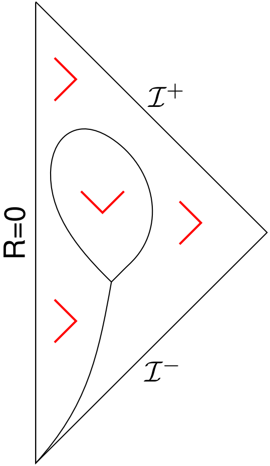
2. arXiv :gr-qc/0504037 (Hayward 2005)

# Contents

- Singularities are often seen as unphysical
  - They won't appear due to quantum effects (J. Bardeen, S. Hayward)
  - Discretization of space-time, noncommutative BHs (P. Nicolini)
- Closed trapping horizons can describe non-singular trapped surfaces
  - Penrose & Hawking singularity theorem (1970) : trapped surface + energy conditions
  - Avoidance of the singularity : the inner horizon does not go to $R \rightarrow 0$




**Formation of a standard BH**




**Formation of a regular BH**

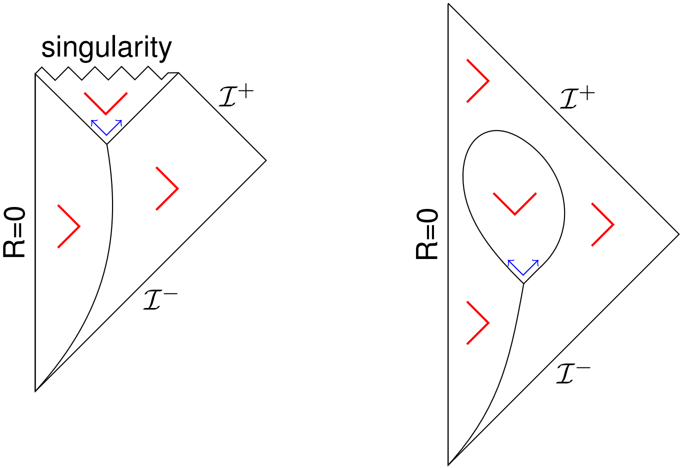
# Contents

# The information loss paradox

- Black holes evaporate (Hawking, 1975) by emitting a thermal spectrum
- This spectrum depends only on $M$, the information on the matter which entered the black hole is lost : no unitary S-matrix can describe the process of formation and evaporation
- One way to solve this issue : closed trapping horizons, from which matter can escape

# The information loss paradox





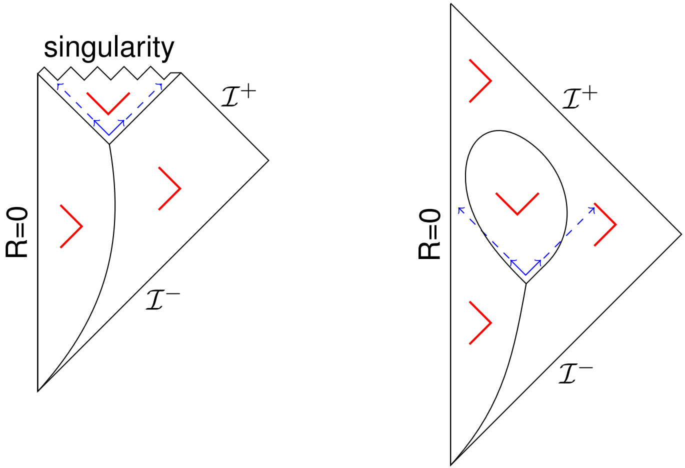

**Formation of a standard BH**

**Formation of a regular BH**

# The information loss paradox

## Information can escape to infinity !






**Formation of a standard BH**

**Formation of a regular BH**

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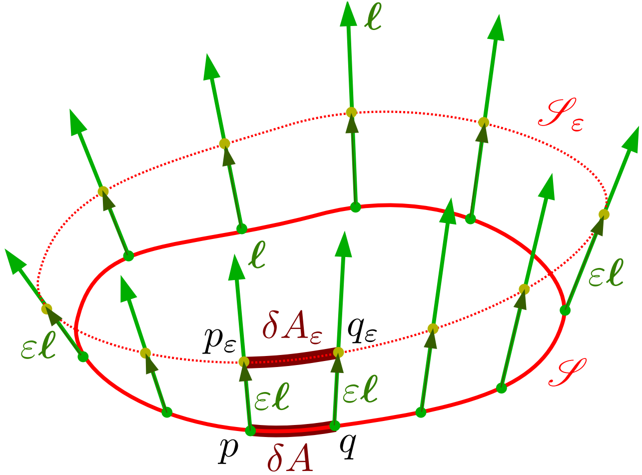
# Expansion of null geodesics [3]

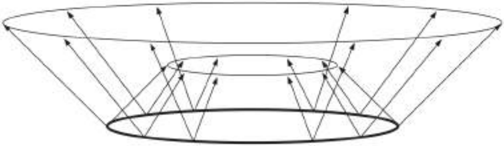



$$\theta_{(l)} = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \frac{\delta A_\epsilon - \delta A}{\delta A} = q^{\mu\nu} \nabla_\mu l_\nu$$

3. figure : E. Gourgoulhon

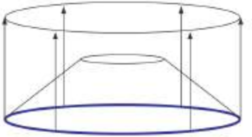
# Expansion of null geodesics

## Notion of apparent/trapping horizon [4]



**Untrapped** surface
$\theta_- < 0, \theta_+ > 0$

**Trapped** surface
$\theta_- < 0, \theta_+ < 0$



**Marginal** surface : apparent horizon
$\theta_- < 0, \theta_+ = 0$

4. arXiv :gr-qc/0504037 (Hayward 2005)

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Motivations
Astrophysical motivation
Absence of singularities
The information loss paradox
Formalism of CTH
The expansion $\theta$
Building models
General conditions
Literature & implementation
Properties
Energy content
NEC violation
Radiation on $\mathcal{I}^+$
Some solutions
Conclusion

# Conditions for the absence of singularities I

$$ds^2 = -F(v,R)dv^2 + 2dvdR + R^2 d\theta^2 + R^2 \sin^2\theta d\phi^2$$

Assume, to recover Vaidya at infinity :

$$F(v,R) = 1 - 2m\frac{R^{\alpha-1} + a_{\alpha-2}R^{\alpha-2} + \cdots + a_1 R + a_0}{R^\alpha + b_{\alpha-1}R^{\alpha-1} + \cdots + b_1 R + b_0}$$

The Ricci and Kretschmann scalars read

$$\begin{cases} \mathcal{R} = g_{\mu\nu}R^{\mu\nu} = -\dfrac{R^2\frac{\partial^2 F}{\partial R^2} + 4R\frac{\partial F}{\partial R} + 2F(v,R) - 2}{R^2} \\ \mathcal{K} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \dfrac{R^4\left(\frac{\partial^2 F}{\partial R^2}\right)^2 + 4R^2\left(\frac{\partial F}{\partial R}\right)^2 + 4F(v,R)^2 - 8F(v,R) + 4}{R^4} \end{cases}$$

# Conditions for the absence of singularities II



Ensure that $\mathcal{R}$ does not diverge at $R = 0$ :

$\Rightarrow$ avoid terms of order 0 et 1 in

$$2\,R\frac{\partial F}{\partial R} + F\,(v,R) - 1 = \frac{-2m(v)}{(R^\alpha + \cdots + b_0)^2}\left[\cdots + R^2\,(5a_2b_0 + a_1b_1 - 3a_0b_2) + R\,(3a_1b_0 - b_1a_0) + a_0b_0\right]$$

$$\Rightarrow b_0 \neq 0 \Rightarrow a_0 = 0 \Rightarrow a_1 = 0 \Rightarrow \boxed{\alpha \geq 3}$$

Finally, minimal form for $F$ :

$$\boxed{F(v,R) = 1 - \frac{2m(v)R^2}{R^3 + 2m(v)b(v)^2}}$$

# Conditions for the formation of CTH

$$\boxed{\theta_+ = \frac{F(v,R)}{R}}$$

$$\theta_+ = 0 \Rightarrow R^3 - 2m(v)R^2 + 2m(v)f(v)^2 = 0$$

Existence of two horizons forming a closed trapped surface (Cardan's method) :

$$m(v) \geq \frac{3\sqrt{3}}{4} b(v)$$

Location of the horizons (positive solutions) :

$$\begin{cases} R_1 &= \frac{4m}{3}\cos\left(\frac{1}{3}\arccos\left(1 - \frac{27b^2}{8m^2}\right)\right) + \frac{2m}{3} \\ R_3 &= \frac{4m}{3}\cos\left(\frac{1}{3}\arccos\left(1 - \frac{27b^2}{8m^2}\right) + \frac{4\pi}{3}\right) + \frac{2m}{3} \end{cases}$$

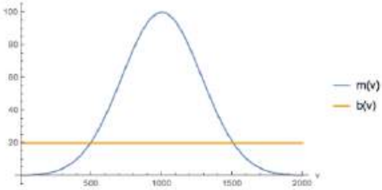
$$\begin{cases} R_1 & \sim 2m - \frac{b^2}{2m} \quad \text{: outer horizon} \\ R_3 & \sim b + \frac{b^2}{4m} \quad \text{: inner horizon} \end{cases}$$

# Contents

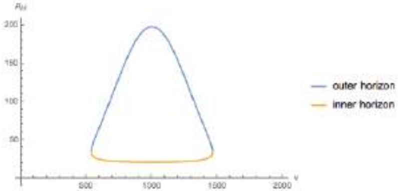
# Literature

- **Hayward :**
  - form a trapped surface at microscopic scale
  - let it evaporate and disappear at microscopic scale
  - $b(v) = \text{cst}$
- **Frolov :**
  - form a trapped surface at microscopic scale
  - let it evaporate and disappear at microscopic scale
  - $b(v) = \text{cst}$
  - implement the Hawking result $\dot{M} \propto -\frac{1}{M^2}$
- **Bardeen :**
  - form a trapped surface at microscopic scale
  - let it evaporate and disappear at macroscopic scale
  - $b(v) \neq \text{cst}$ : bounce of the inner horizon (covariant entropy bound)

# Hayward-like model




(a) Mass

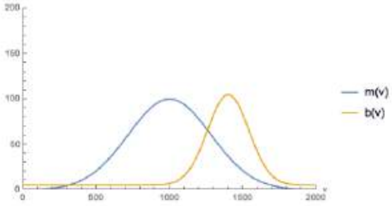



(b) Outer and inner apparent horizons

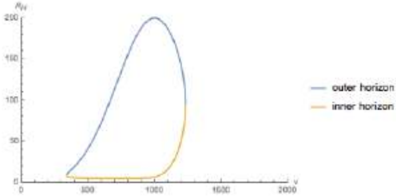
# Implementation : explicit models

## Bounce of the inner horizon

**Bardeen-like model**




(a) Mass




(b) Outer and inner apparent horizons

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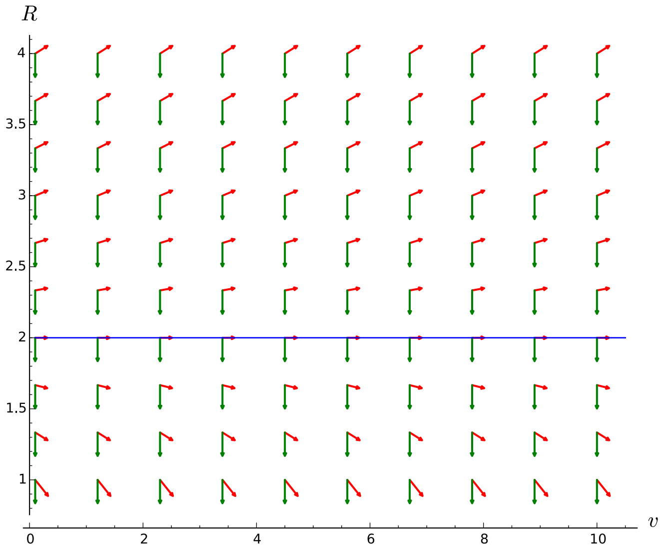
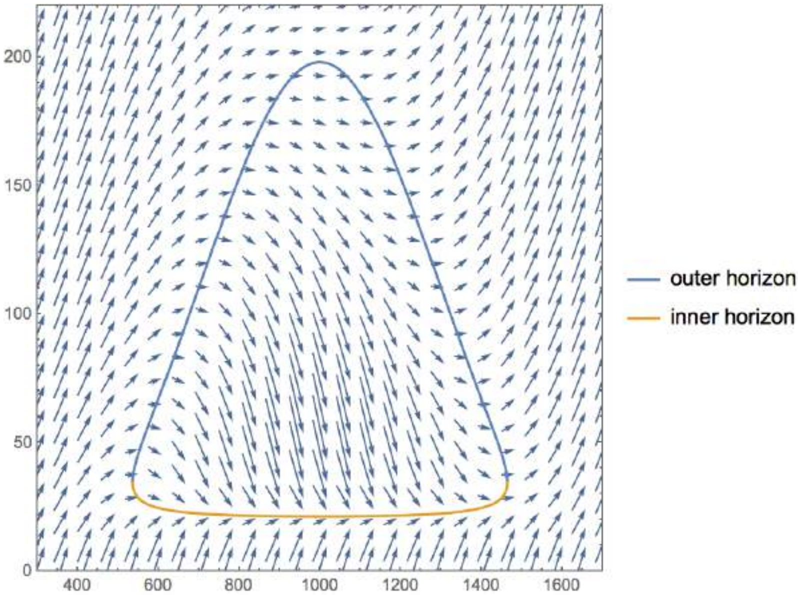

R
4
3.5
3
2.5
2
1.5
1
0
2
4
6
8
10
v

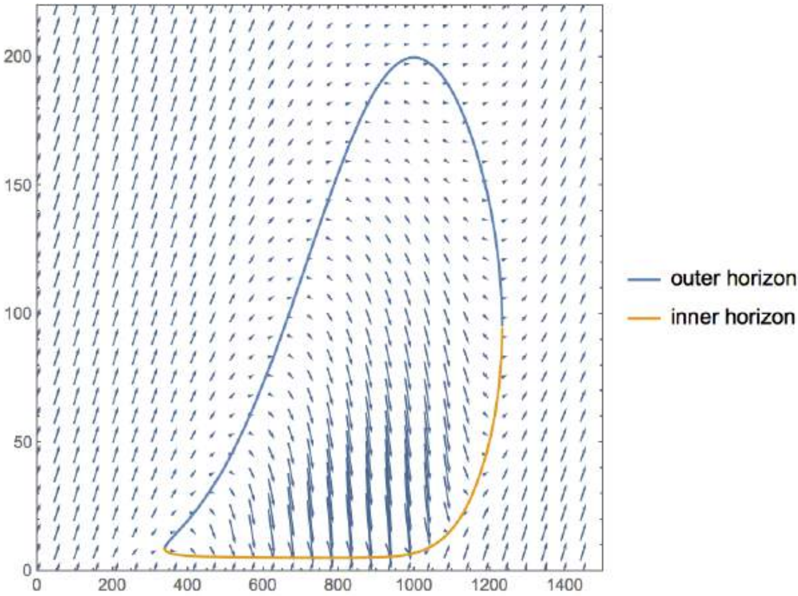
# Geodesics

## Hayward-like model



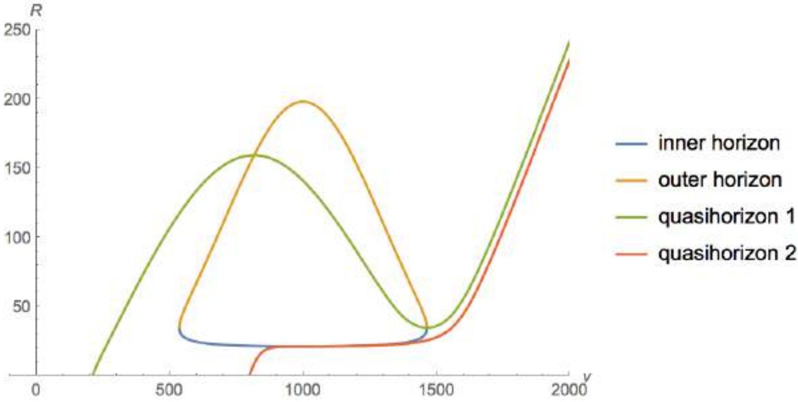
# Geodesics

## Bardeen-like model



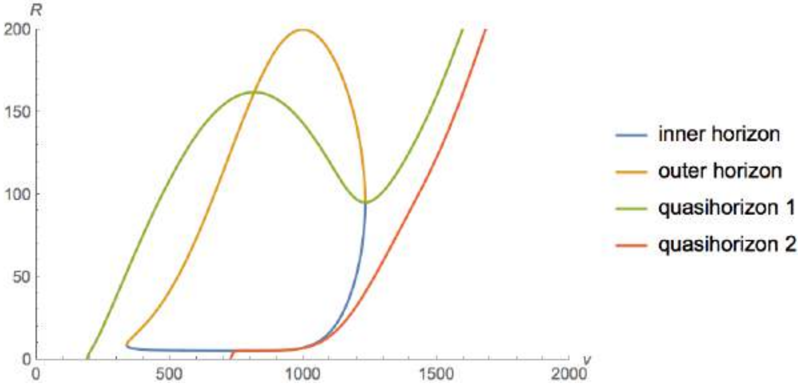
# Quasi-horizons

## Hayward-like model



# Quasi-horizons
## Bardeen-like model





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# Violation of the null energy condition

$$\boxed{T_{\mu\nu}k^\mu k^\nu \geq 0}$$

Take a general null vector $k^\mu = (\gamma, \delta, 0, 0)$ which thus satisfies :

$$0 = k_\mu k^\mu = -F\gamma^2 + 2\gamma\delta$$

The NEC then reads :

$$T_{\mu\nu}k^\mu k^\nu = \gamma^2 T_{vv} + 2\gamma\delta T_{vR} \geq 0$$

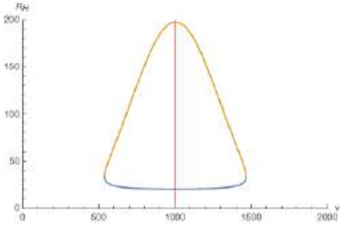
By virtue of Einstein's equations, one gets

$$\begin{cases} 8\pi T_{vv} & = -\frac{\left(F(v,R)\frac{\partial F}{\partial R}+\frac{\partial F}{\partial v}\right)R+F(v,R)^2-F(v,R)}{R^2} \\ 8\pi T_{vR} & = \frac{R\frac{\partial F}{\partial R}+F(v,R)-1}{R^2} \end{cases}$$

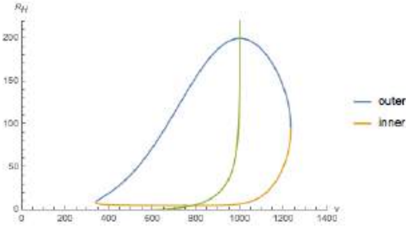
Finally :

$$
\begin{aligned}
T_{\mu\nu}k^\mu k^\nu &= \gamma^2 T_{vv} + 2\gamma\delta T_{vR} \\
&= \left(-FT_{vR} - \frac{\partial_v F}{8\pi R}\right) + 2\gamma\delta T_{vR} \\
&= (-F\gamma^2 + 2\gamma\delta)T_{vR} - \frac{\gamma^2 \partial_v F}{8\pi R} \\
&= -\frac{\gamma^2 \partial_v F}{8\pi R}
\end{aligned}
$$

$$
\boxed{\text{NEC violation} \Leftrightarrow \partial_v F > 0}
$$




Hayward-like model




Bardeen-like model

$\Rightarrow$ the NEC is violated up to infinity !

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# Radiation on $\mathcal{I}^+$

Effective approach :

$$ds^2 = -F(v,R)dv^2 + 2dvdR + R^2 d\theta^2 + R^2 \sin^2\theta d\phi^2$$

$$G_{\mu\nu} = 8\pi < T_{\mu\nu} >$$

$\Rightarrow$ the Hawking radiation on $\mathcal{I}^+$ must be contained from the start in the chosen geometry.

$\Rightarrow$ on $\mathcal{I}^+$, one expects an energy-momentum tensor of the outgoing Vaidya type :

$$T_{\mu\nu} = -\frac{\partial_u m}{4\pi R^2} k_\mu k_\nu$$

For a spherically symmetric spacetime :

$$
\begin{aligned}
T =& \alpha k \otimes k + \beta l \otimes l + \gamma \left(k \otimes l + l \otimes k\right) \\
& + \delta \left(d\theta \otimes d\theta + \sin^2\theta d\phi \otimes d\phi\right)
\end{aligned}
$$

Such a tensor can also be decomposed onto an orthonormal frame :

$$
\begin{aligned}
T =& T^{\hat{0}\hat{0}} e_{\hat{0}} \otimes e_{\hat{0}} + T^{\hat{0}\hat{1}} e_{\hat{0}} \otimes e_{\hat{1}} + T^{\hat{1}\hat{0}} e_{\hat{1}} \otimes e_{\hat{0}} + T^{\hat{1}\hat{1}} e_{\hat{1}} \otimes e_{\hat{1}} \\
& + T^{\hat{2}\hat{2}} \left(e_{\hat{2}} \otimes e_{\hat{2}} + e_{\hat{3}} \otimes e_{\hat{3}}\right)
\end{aligned}
$$

Define two independant null vectors :

$$
\begin{cases}
k' = \dfrac{e_{\hat{0}} - e_{\hat{1}}}{\sqrt{2}} \\
l' = \dfrac{e_{\hat{0}} + e_{\hat{1}}}{\sqrt{2}}
\end{cases}
$$

$$ds^2 = -F(v,R)dv^2 + 2dvdR + R^2 d\theta^2 + R^2 \sin^2\theta d\phi^2$$

$$\Rightarrow \begin{cases} \alpha \propto \left(T^{\hat{0}\hat{0}} - 2T^{\hat{0}\hat{1}} + T^{\hat{1}\hat{1}}\right) = 0 \\ \beta \propto \left(T^{\hat{0}\hat{0}} + 2T^{\hat{0}\hat{1}} + T^{\hat{1}\hat{1}}\right) = -\frac{4\frac{\partial F}{\partial v}}{RF(v,R)} \\ \gamma \propto \left(T^{\hat{0}\hat{0}} - T^{\hat{1}\hat{1}}\right) = -4\frac{R\frac{\partial F}{\partial R} + F(v,R) - 1}{2R^2} \end{cases}$$

$$ds^2 = -F(v,R)dv^2 + 2dvdR + R^2 d\theta^2 + R^2 \sin^2\theta d\phi^2$$

$$\Rightarrow \begin{cases} \alpha \propto \left(T^{\hat{0}\hat{0}} - 2T^{\hat{0}\hat{1}} + T^{\hat{1}\hat{1}}\right) = 0 \\ \beta \propto \left(T^{\hat{0}\hat{0}} + 2T^{\hat{0}\hat{1}} + T^{\hat{1}\hat{1}}\right) = -\frac{4\frac{\partial F}{\partial v}}{RF(v,R)} \\ \gamma \propto \left(T^{\hat{0}\hat{0}} - T^{\hat{1}\hat{1}}\right) = -4\frac{R\frac{\partial F}{\partial R} + F(v,R) - 1}{2R^2} \end{cases}$$

Taking $F = 1 - \frac{2m(v)}{R}$ :

$$\begin{cases} \alpha = 0 \\ \beta \propto \frac{4\partial_v m}{R^2 F} \\ \gamma = 0 \end{cases}$$

$$ds^2 = -F(v,R)dv^2 + 2dvdR + R^2 d\theta^2 + R^2 \sin^2\theta d\phi^2$$

$$\Rightarrow \begin{cases} \alpha \propto \left(T^{\hat{0}\hat{0}} - 2T^{\hat{0}\hat{1}} + T^{\hat{1}\hat{1}}\right) = 0 \\ \beta \propto \left(T^{\hat{0}\hat{0}} + 2T^{\hat{0}\hat{1}} + T^{\hat{1}\hat{1}}\right) = -\frac{4\frac{\partial F}{\partial v}}{RF(v,R)} \\ \gamma \propto \left(T^{\hat{0}\hat{0}} - T^{\hat{1}\hat{1}}\right) = -4\frac{R\frac{\partial F}{\partial R} + F(v,R) - 1}{2R^2} \end{cases}$$

Taking $F = 1 - \frac{2m(v)}{R}$ :

$$\begin{cases} \alpha = 0 \\ \beta \propto \frac{4\partial_v m}{R^2 F} \\ \gamma = 0 \end{cases}$$

Ingoing Vaidya solution : $T_{\mu\nu} = \frac{\partial_v m}{4\pi R^2} l_\mu l_\nu$ ✓

$$ds^2 = -F(v,R)dv^2 + 2dvdR + R^2 d\theta^2 + R^2 \sin^2\theta d\phi^2$$

$$\Rightarrow \begin{cases} \alpha \propto \left(T^{\hat{0}\hat{0}} - 2T^{\hat{0}\hat{1}} + T^{\hat{1}\hat{1}}\right) = 0 \\ \beta \propto \left(T^{\hat{0}\hat{0}} + 2T^{\hat{0}\hat{1}} + T^{\hat{1}\hat{1}}\right) = -\frac{4\frac{\partial F}{\partial v}}{RF(v,R)} \\ \gamma \propto \left(T^{\hat{0}\hat{0}} - T^{\hat{1}\hat{1}}\right) = -4\frac{R\frac{\partial F}{\partial R} + F(v,R) - 1}{2R^2} \end{cases}$$

Taking $F = 1 - \frac{2m(v)}{R}$ :

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Ingoing Vaidya solution : $T_{\mu\nu} = \frac{\partial_v m}{4\pi R^2} l_\mu l_\nu$ ✓

$\alpha = 0 \Rightarrow$ impossible to get an outgoing Vaidya solution !

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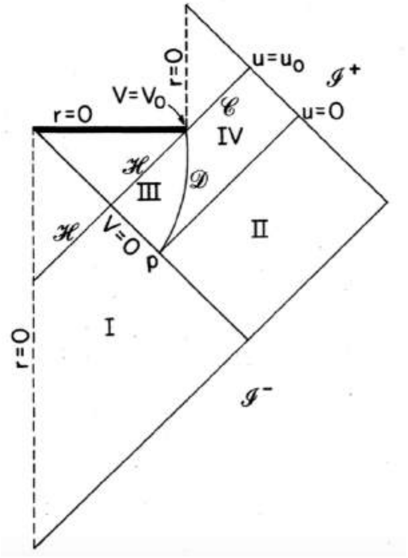
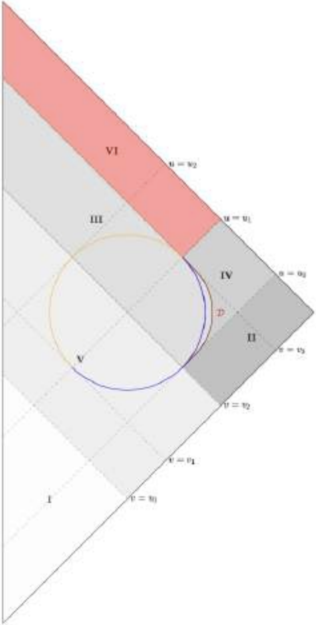
# Making junctions (Hiscock, 1981)



# Using a more general metric

## Bardeen 2014, Page 2016

Most general spherically symetric spacetime :

$$ds^2 = -e^{2\psi(v,R)}F(v,R)dv^2 + 2e^{\psi(v,R)}dvdR + R^2d\Omega^2$$

Page (2016) showed that for some choice of $F$ and $\psi$ one can reduce this metric to an outgoing Vaidya metric, and thus describe a radiation flux.

$\Rightarrow$ implementation of this method :

- one could use a single metric on the whole spacetime
- it would not correspond in general to known matter
- but the stress-energy tensor would behave as expected in the right limits ($\mathcal{I}^-$, $\mathcal{I}^+$)

# Using a more general metric

For this choice of metric :

$$\begin{cases} \alpha \propto \dfrac{F(v,R)\frac{\partial \psi}{\partial R}}{2R} \\ \beta \propto \dfrac{\left(F^2 e^{\psi}\frac{\partial \psi}{\partial R} - 2\frac{\partial F}{\partial v}\right)e^{-\psi}}{2RF(v,R)} \\ \gamma \propto -\dfrac{RF\frac{\partial \psi}{\partial R} + R\frac{\partial F}{\partial R} + F - 1}{2R^2} \end{cases}$$

The NEC reads : $T(k,k) = \beta \geq 0$, $T(l,l) = \alpha \geq 0$

The constraints on $\psi$ are then given by :

| $\mathcal{I}^-$ | $\mathcal{I}^+$ |
|---|---|
| $\beta \gg \alpha, \gamma, \delta$ | $\alpha \gg \beta, \gamma, \delta$ |
| $\alpha, \beta \geq 0$ | $\alpha, \beta \geq 0$ |

# Conclusion

- We have given general conditions as well as explicit construction for models describing the formation and evaporation of a non-singular trapped surface
- To this end, we used the formalism of closed trapping horizons developed by Hayward
- This approach is effective, in the sense that we start from a metric containing quantum effects and solve $G_{\mu\nu} = 8\pi < T_{\mu\nu} >$ to analyse the energy content
- The analysis of the energy-momentum tensor gives some conditions on the function $\psi$, which still has to be obtained
- The radiation of the inner horizon, which is not a resolved question, could explain the absence of singularity