Black Hole fusion made easy

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A recent development in General Relativity with very old roots

Work with Marina Martínez
CQG 33, 155003 (2016) arXiv:1603.00712
IJMP D25, 1644015 (2016)

and with
Marina Martínez & Miguel Zilhão
to appear
Black Hole fusion

The most complex of all processes governed by $R_{\mu\nu} = 0$

Non-linearity at its most fiendish
Black Hole fusion

The most complex of all processes governed by \( R_{\mu\nu} = 0 \)

Non-linearity at its most fiendish

or maybe not—not always
This is what we’d see (lensing)

Not a black hole, but its shadow
What is a black hole?

Spacetime region from which not even light can escape

Event Horizon
Star
Spherical wavefronts contract, then expand
Collapsed Star

singularity
outgoing light ray escapes

outgoing light ray doesn’t escape

light ray separatrix
EVENT HORIZON
Null hypersurface

3-dimensional in 4-dimensional spacetime
Null hypersurface made of null geodesics (light rays)
caustic

where null geodesics enter to form part of event horizon
Event horizon
found by
tracing a family of light rays in a given spacetime
Event horizon of binary black hole fusion
Event horizon of binary black hole fusion

“pants” surface

\[ t \]

Light rays that form the EH
Event horizon of binary black hole fusion

Cover of *Science*, November 10, 1995

*Binary Black Hole Grand Challenge Alliance* (Matzner et al)
**Spatial sections** of event horizon of binary black hole fusion


Surely the fusion of horizons can only be captured with supercomputers.
Surely the fusion of horizons can only be captured with supercomputers

or so it’d seem
∃ limiting (but realistic) instance where horizon fusion can be described exactly

It involves only elementary ideas and techniques
Equivalence Principle (1907)

Schwarzschild solution & Null geodesics (1916)

Kerr solution (1964)

Notion of Event Horizon (1950s/1960s)
Extreme-Mass-Ratio (EMR) merger

\[ m \ll M \]
\( m \ll M \)

most often taken as

\[ m \to 0 \]

\( M \) finite

\( M \) sets the scale for the radiation emitted
Fusion of horizons involves scales $\sim m$.
Gravitational waves?

When $M \to \infty$ the radiation zone is pushed out to infinity

No gravitational waves in this region
Gravitational waves?

GWs will reappear if we introduce corrections for finite small $\frac{m}{M}$

matched asymptotic expansion to Hamerly+Chen 2010
Hussain+Booth 2017

not for today
\( M \to \infty \)
Very large black hole / Very close to the horizon
Very close to a Black Hole

Horizon well approximated by null plane in Minkowski space
This follows from the **Equivalence Principle**

At short enough scales, geometry is equivalent to flat Minkowski space.

Curvature effects become small, but horizon remains...
Locally gravity is equivalent to acceleration

Locally black hole horizon is equivalent to acceleration horizon
Falling into very large bh = crossing a null plane in Minkowski space
Object falling into a Large Black Hole

in rest frame of infalling object
Small Black Hole falling into a Large Black Hole in rest frame of small black hole
Small Black Hole falling into a Large Black Hole

both are made of lightrays
Lightrays must merge to form a pants-like surface

“oversized leg”

“thin leg”
How?

EH is a family of light rays in spacetime.

Small black hole: Schwarzschild solution with finite mass \( m \).
To find the pants surface:

Trace a family of null geodesics in the Schwarzschild solution that approach a null plane at infinity
All the equations you need to solve

\[ t_q(r) = \int \frac{r^3 \, dr}{(r-1)\sqrt{r(r^3-q^2(r-1))}} \]

\[ \phi_q(r) = \int \frac{q \, dr}{\sqrt{r(r^3-q^2(r-1))}} \]

with appropriate final conditions:

null plane at infinity

\[ 2m = 1 \]

q = impact parameter of light rays at infinity
Null geodesics in Schwarzschild solution

light rays asymptoting to a plane at infinity

Schwarzschild horizon
Null geodesics in Schwarzschild solution

light rays asymptoting to a plane at infinity

simply, integrate back in time
Null geodesics in Schwarzschild solution

light rays asymptoting to a plane at infinity

simply, integrate back in time
“Pants” surface

big black hole

small black hole

big black hole
Sequence of constant-time slices

\[ t = -20r_0 \]

\[ t = -10r_0 \]

\[ t = -2r_0 \]

\[ t = 0 \] pinch-on

\[ t = r_0 \]

\[ t = 6r_0 \]

\[ t = -0.1r_0 \]

\[ t = 27r_0 \]

\[ r_0 = \text{small horizon radius} \]
Preferred time-slicing

∃ timelike Killing vector
Schwarzschild time

Rest-frame of small black hole is well defined
Complete characterization of merger

*Precise quantitative* results for:

- Elongation of horizon at pinch
- Duration of fusion
- Line of caustics
- Area increase
- Critical behavior at pinch
- Simple local model for pinch
The full monty

The ultimate description of EMR mergers
Rotation and motion

Large black hole rotation
Relative motion in infall

Just a boost
Equivalent to a rotation of the surface
Small black hole rotation

Change Schwarzschild $\rightarrow$ Kerr

Fusion of any EMR Black Hole binary in the Universe

to leading order in $\frac{m}{M} \ll 1$
\frac{a}{M} = 0.8
\[ \frac{a}{M} = 0.9 \]
Final remarks
Simple, accurate, generic description of a process that is happening all over the Universe
Can we *observe* this?

Maybe not

Then, what is it good for?
Fusion of Black Hole Event Horizons is a signature phenomenon of General Relativity.

Equivalence Principle allows to capture and understand it easily in a (realistic) limit.
Exact construction

Benchmark for detailed numerical studies

First step in expansion in $\frac{m}{M} \ll 1$

to incorporate gravitational waves

(matched asymptotic expansion)
Equivalence Principle magic

Get 2 black holes
out of a geometry with only 1

This could have been done (at least) 50 years ago!
Gravitational waves?

Quasinormal vibrations

wavelength $\sim M$ : become constant

wavelength $\sim m$ : $\ell \sim \frac{M}{m} \gg 1$

localized near photon orbit at distance $\sim M \rightarrow \infty$

No gravitational waves in this region
Elongation at pinch-on

\[ r_0 = 2m = \text{radius of small bh} \]
\[ r_* = \text{radius at pinch-on point} \]

\[ r_* = 1.76 \, r_0 \]

\( r_* = 1.76 \, r_0 \)
Duration of fusion

\[ t = 5.95 \, r_0 \]
Area increase

From null-generator expansion
\[ \frac{\Delta A_{\text{smallbh}}}{A_{\text{smallbh}}} = 0.242 \]

with null-generator addition
\[ \frac{\Delta A_{\text{smallbh}}}{A_{\text{smallbh}}} = 0.794 \]

(total \( \Delta A \approx 32\pi Mm \rightarrow \infty \) from added generators)
Pinch-on: Criticality

Opening angles of cones $\sim |t|^{1/2}$

$\exists$ simple local model for pinch
Pinch-on: Criticality

Throat growth $\sim t$

$\exists$ simple local model for pinch